RING STRUCTURE OF BEAMS ACCOMPANYING SELF-ACTION OF ELECTROMAGNETIC WAVES IN A PLASMA

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It is well known that the self-action of electromagnetic waves in a plasma is associated with the change induced in the plasma density by thermal heating. In a weakly ionized plasma this change in the density depends entirely on the flow of electrons out of the more strongly heated region and on the increase in the density owing to ionization of atoms. For different plasma parameters and parameters of the incident radiation, one or another mechanism for the change in the density predominates, which substantially alters the dynamics of wave propagation from the case of a completely ionized plasma.

In this paper we present the results of a numerical study of the self-action of millimeter-range electromagnetic waves in a low-temperature plasma whose parameters are close to those of a gas-discharge plasma. Specific calculations are carried out for a nitrogen plasma.

In this case, the self-action of electromagnetic waves in the plasma is described by the following system of equations [1-7]:

$$\frac{\partial n}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}run=\alpha(N_h-n)n-\beta n^3-\beta_\nu n^2, \tag{1}$$

$$\frac{\partial}{\partial t}\left(\frac{3}{2}nT_e\right)=\sigma n|E|^2-\frac{3}{2}\frac{2m}{M}(T_e-T_h)(\nu_{ei}+\nu_{ea})n-$$

$$-I^+(\alpha(N_h-n)-\beta n^2)n-I^*(\gamma(N_h-n)-\gamma^*)n+\frac{1}{r}\frac{\partial}{\partial r}rq_e\frac{\partial T_e}{\partial r},$$

$$\frac{\partial}{\partial t}\left(\frac{3}{2}N_hT_h\right)=\frac{3}{2}\frac{2m}{M}(T_e-T_h)(\nu_{ei}+\nu_{ea})n,$$

$$2ik\frac{\partial E}{\partial z}+\Delta_\perp E+\frac{4\pi}{c^2}\sigma nE\omega(i+\omega/\nu)=0,$$

$$\nu=\nu_{ei}+\nu_{ea},\ \sigma=\frac{e^2}{m}\frac{\nu}{\omega^2+\nu^2},\ q_e=\frac{5}{2}\frac{nT_e}{m\sum_k\nu_{ek}},$$

$$\mathcal{E}=\frac{1}{\sqrt{2}}\left(Ee^{-i\omega t}+\text{c.c}\right).$$

where $\mathcal{E}$ is the intensity of the electric field; n and N_h, density of the electronic and heavy components of the plasma; T_e and T_h, temperature of the electrons and heavy particles; ν_{ei}, ν_{ea}, electron–ion and electron–atom collision frequencies; α, ionization factor; β, three-particle recombination factor; β_ν, coefficient of photorecombination; γ, coefficient of excitation of atoms; q_e, coefficient of electronic thermal conductivity. The diffusion velocity is given by [6]

$$u=-D_a\nabla n/n-D_e\nabla T_e/T_e-D_i\nabla T_i/T_i, \tag{2}$$

where $D_a = D(1 + T_e/T_i)$; $D_e = DT_e/2T_i$; $D_i = D/2$; $D = 2T_i/M\nu_{ia}$; ν_{ia} is the ion–atom collision frequency.

The kinetic coefficients are [2-6, 8, 9]:

$$\nu_{ei}=\frac{4\pi ne^4\ln\Lambda}{\sqrt{\frac{27}{4}mT^3}},\quad \nu_{ea}=\frac{4}{3}\sigma_{ea}\sqrt{\frac{8T_e}{\pi m}}(N_h-n),$$

$$\alpha=1.7\cdot10^{-5}\frac{T_e}{(I^++T_e)^{1/2}I^+(I^++0.21T_e)}e^{-\frac{I^+}{T_e}}\ [\text{cm}^3\cdot\text{sec}^{-1}],$$

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$$\beta = 1.5\cdot 10^{-27} T_e^{-9/2} \ [\text{cm}^6\cdot\text{sec}^{-1}], \qquad \beta_v = 2.7\cdot 10^{-13} T_e^{-3/4} \ [\text{cm}^3\cdot\text{sec}^{-1}].$$

It is assumed that for the chosen parameters the temperation of the ions T_i and the temperature of the atoms T_a coincide: $T_i = T_a = T_h$), and that the bremmstrahlung is insignificant. Reabsorption in atomic lines is ignored, i.e., it is assumed that resonance radiation escapes the plasma $\gamma^* = 0$. Derivatives with respect to the longitudinal coordinate z are dropped in the equations of plasma dynamics. This is justified, since the characteristic scale of variation of the variables in the radial direction is much smaller than that in the direction of propagation $z_0 \sim (kr_0)r_0$, $kr_0 >> 1$.

It should be noted that the numerical values of the kinetic coefficients presented in different sources differ substantially. This concerns especially the kinetic coefficients of inelastic processes, the spread in whose values reaches several orders of magnitude. Hence there is a substantial arbitrariness in the numerical values of most kinetic coefficients.

Using the system (1) we shall study the propagation of a Gaussian electromagnetic pulse in a plasma, i.e., at the boundary of the plasma z = 0

$$E = E_0 \exp\left(-r^2/r_0^2 + t^2/\tau_0^2\right).$$

The dynamics of the pulse is studied as a function of the amplitude of the electric-field intensity of the wave E_0.

Results were obtained for the following values of the parameters: $n = 2\cdot 10^{13}$ cm^{-3}, $N_h = 10^{16}$ cm^{-3}, $T_e = T_h = 0.3$ eV, $\lambda = 0.5$ cm, M = 14, $r_0 = 2$ cm, $\tau_0 = 4\cdot 10^{-4}$ sec. For these values, the following regions of values of E_0, where the nature of the wave propagation is qualitatively different, can be singled out: $E_0 < 50$ V/cm, 50 V/cm $< E_0 <$ 200 V/cm, 200 V/cm $< E_0 <$ 300 V/cm, 300 V/cm $< E_0 <$ 450 V/cm.

Up to amplitudes $E_0 < 50$ V/cm the nonlinear effects are small. For $E_0 > 50$ V/cm, self-interaction effects appear. This is linked with the fact that in the radial direction the plasma is heated nonuniformly, as a result of which thermal diffusion arises from the more heated region on the axis of the pulse r = 0. Thermal diffusion is determined by the second term in the expression for the diffusion velocity (2), since in the region $E_0 \geqq 50$ V/cm the estimates $\nabla n/n << \nabla T_e/T_e$, $\nabla T_i/T_i << \nabla T_e/T_e$ hold. As a result, a minimum of the electron density ("density well") forms on the axis. This distribution of the electron density acts on the wave as a focusing lens, which gives rise to self-focusing of the electromagnetic wave. The critical value of the derivation of the electron density from the initial value on the axis r = 0, at which self-focusing arises, constitutes $\delta n \sim 0.04 n_0$, where $n_0 = 2\cdot 10^{13}$ cm^{-3}. The electron density in this case $T_e \leqq 0.4$ eV. The ion temperature differs little from the initial temperature $T_h \cong 0.33$ eV. As E_0 increases, the maximum value of δn increases up to $\delta n \sim 0.2 n_0$ at $E_0 \sim 150$ V/cm, and then decreases. This is determined by the increase in the diffusion spreading of the "density well" [the first term in the expression (2) for the diffusion velocity] and by the inclusion of the ionization of atoms. Naturally, as δn decreases, the effectiveness of self-focusing decreases, i.e., the maximum intensity of the electric field of the wave that can be achieved in the process of self-focusing decreases. Up to intensities $E_0 \sim 200$ V/cm the electron temperature T_e does not exceed 0.8 eV and inelastic processes still have virtually no effect on the dynamics of the plasma, so that the evolution of the pulse here coincides qualitatively with the typical picture of self-focusing in media with a cubic nonlinearity.

For intensities $E_0 > 200$ V/cm the electron temperature $T_e > 0.8$ eV and the ionization of atoms becomes important. The growth in the electron density owing to ionization near the axis exceeds the electron diffusion, the electron density peaks on the axis of the pulse, and the minimum of the density shifts toward the edge of the beam. This causes the field to be displaced out of the region near the axis and a maximum of the field to appear at the edge. In the radial section the distribution of the radiation intensity acquires a ring-shaped form, i.e., self-limitation of the field on the axis occurs [10, 11] and ring-shaped self-focusing appears. Since the process develops in time, the plasma is heated nonuniformly in the direction of propagation of the wave also. The plasma is heated more strongly in the region near the boundary z = 0 and less strongly on the leading edge of the pulse, where there is always a region with low ionization of atoms; in this region the minimum of the electron density is located on the axis r = 0, i.e., at a given point z on the axis of the pulse there at first forms a minimum of the electron density, since the diffusion outflow of electrons from the region near the axis still predominates. Then as the electron temperature increases

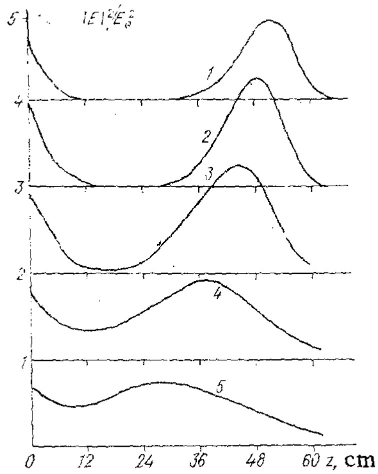

Fig. 1

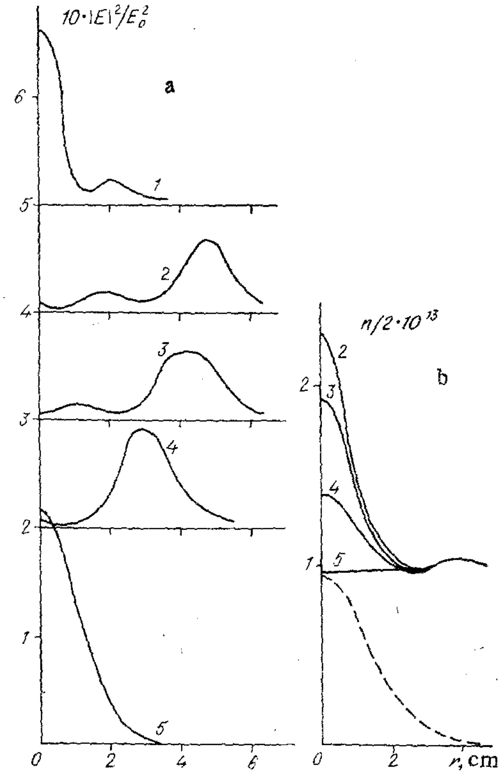

Fig. 2

the ionization of atoms by electrons is switched on. When the growth in the electron density owing to ionization exceeds the diffusion outflow, the minimum is displaced along the radius and the electron density peaks on the axis. This distribution of the density acts like a defocusing lens, displacing the electromagnetic wave out of the region r = 0. As a result the peak of the intensity is displaced away from the point r = 0. For this reason for a quite long time there exist regions with the usual self-focusing with a maximum on the axis (on the leading front of the pulse) and a region of ring-shaped self-focusing in the boundary region of the plasma. It is natural that at the initial stage only the usual self-focusing is realized.

Thus the spatial distribution of the field intensity looks like a hollow closed body with a moving leading edge. The typical axial distribution of the intensity for 200 V/cm < E_0 < 450 V/cm is shown in Fig. 1, where the horizontal lines show the zero level of the field for each t, the lines 1-5 correspond to t = 290, 260, 230, 190, and 170 μsec, E_0 = 310 V/cm. The leading maximum is the region of the usual self-focusing. This phenomenon is also observed with a high degree of ionization, when the hydrodynamic motion of the plasma is significant.

For E_0 > 300 V/cm the axial distribution of the intensity does not qualitatively differ from that of the preceding case, but in the radial direction the picture becomes more complicated, since the boundary layer of the plasma is heated more strongly and the electron density peaks there. Since the electron temperature at this point increases proportionally to the square of the field intensity and is proportional to the electron density, as follows from the first equation of the system (1), the density peak has sharp gradients with respect to r and z and the characteristic transverse size is less than the characteristic size of the wave. Diffraction of the wave – self-diffraction of the electromagnetic wave – occurs on this radiation-induced peak in the density. The oscillations of the intensity formed by the self-diffraction give rise to the appearance of a multiring structure in the transverse section of the wave. On the leading edge of the pulse these oscillations can be encompassed by the density well and be pressed toward the axis in the region of the usual self-focusing. The oscillations are quite weak; the first peak closest to the axis is most clearly visible. Figure 2 (E_0 = 420 V/cm) shows the dynamics of the formation of structure in the radial distribution of the radiation intensity (a) and the density on the plasma boundary z = 0 (b). Line 5 in Fig. 2a corresponds to t = 90 μsec and z = 16 cm, and the electron density still has a minimum on the axis; lines 4-2 (t = 170, 185, and 190 μsec, z = 16 cm) is the region of ring-shaped self-focusing; line 1 is the region of ordinary self-focusing (z = 42 cm). The broken line in Fig. 2b shows the radial distribution of the intensity of the electromagnetic wave $|E|/E_0$ at z = 0.

We shall estimate the magnitude of the density peak required for the appearance of interference maxima. For this, the difference in the path lengths of the rays from the center of the beam edge at the thickness of the peak ℓ should attain the value of the wavelength λ:

$$(\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2})\, l \sim \lambda, \quad \frac{4\pi e^2}{m\omega^2}\, \delta n l \sim 2\lambda,$$

$$\delta n \sim \frac{2\pi c^2 m}{e^2 l\lambda} \sim 2.3 \cdot 10^{13}/l\lambda \,[\mathrm{cm}^{-3}].$$

As is evident from this estimate, this phenomenon of self-diffraction of waves is most clearly observed in the long-wavelength region of the spectrum. Thus, for λ = 0.5 μm, ℓ = 0.1 cm the nonuniformity of the plasma density over the width of the beam $\delta n \sim 5 \cdot 10^{18}$ cm^{-3}. In our case, λ = 0.5 cm, $\ell \sim 4$ cm (from the numerical calculations), $\delta n \sim 10^{13}$ cm^{-3}.

This ring-shaped structure in the distribution of the radiation intensity naturally gives rise to a ring-shaped structure in the distribution of the electron temperature. In the calculations these ring-shaped nonuniformities of the temperature were weak and did not exceed $\delta T \sim 0.1 T_e$. A ring structure did not arise in the distribution of the temperature of the heavy component of the plasma.

For E_0 > 450 V/cm, the growth in the electron density owing to ionization of atoms already at the initial stage of the process is much faster than the diffusion outflow. A peak in the electron density forms on the axis, and self-focusing does not arise.

Similar calculations were carried out for focused (defocused) beams. The angle of divergence varied from φ = +30 to φ = −30. For low field intensities (E_0 < 200 V/cm) changing the angle of divergence merely results in insignificant quantitative changes. For E_0 > 200 V/cm the dynamics of the pulse depends on the relations between the angle φ and the angle of self-focusing φ_{sf}. If the absolute value of φ is less than that of φ_{sf}, then the dynamics of the wave is virtually independent of the value of φ. If the absolute value of φ exceeds that of φ_{sf}, then self-focusing does not arise at all, which is natural for diverging beams; for converging beams this is linked with the fact that the density peak forming at the focal point acts like a defocusing lens.

Equations (1) do not take into account the hydrodynamic motion of the gas. The following estimate can be obtained for the hydrodynamic velocity of the gas from the second and third equations of the system (1):

$$v_g \sim \frac{\tau_0}{r_0}\frac{\nabla p}{MN_h}, \quad p \sim \tau_0 \frac{e^2}{m}\frac{\nu}{\omega^2} n |E|^2, \quad \omega \gg \nu,$$

whence $p \sim \mathrm{const}\, n^2 |E|^2/T_e^{3/2}$.

In the radial direction the value of $|E|^2$ drops while that of $T_e^{-3/2}$ increases. This dependence of the pressure on the temperature, density, and intensity leads to the fact that for the gradient of the total pressure the estimate $\nabla p \ll p/r_0$ is correct. As the numerical calculations with the specific values of the parameters presented show, the diffusion velocity is 2-3 times higher than the hydrodynamic velocity. Naturally, it makes sense to retain the leading terms, since the numerical values of the kinetic coefficients are determined with a large degree of arbitrariness. Because of the strong dependence of the pressure on the density $\sim n^2$, however, already for $n = 4 \cdot 10^{13}$ cm^{-3}, $N_h = 10^{16}$ cm^{-3} the hydrodynamic velocity predominates. At the same time, for $n = 2 \cdot 10^{13}$ cm^{-3}, $N_h = 10^{16}$ cm, E_0 = 420 V/cm, taking into account the hydrodynamic motion of the gas gives only small quantitative changes $\delta N_h/N_h < 0.1$.

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TRANSFORMATION OF THE BENNET DISTRIBUTION IN A RAREFIED GAS

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The usual Bennet distribution of particles in a beam is characterized by a density which decreases comparatively slowly with increasing radius; this is a negative factor in the solution of problems associated with the formation of a thin, powerful, quasistationary, relativistic electron beam (REB). Under sufficiently high pressure of the residual gas the effect of the current of secondary electrons of the plasma, produced by the ionization of the gas by the beam, on the state of the main beam must be taken into account. As a result of this effect, an equilibrium state of the REB with a steeper, than in the case of the Bennet distribution, drop in the particle density toward the periphery of the beam can form.

For sufficiently high currents in the REB ($I \geqq 1$ kA) the average Larmor radius of the electrons of the secondary plasma is shorter than the mean-free-path length λ_0 (magnetized diffusion) and the flow of secondary electrons acquires, owing to the magnetization, a longitudinal component [1]. The secondary flow can change the parameters of the REB, if the secondary-electron current is comparable to the beam current. In what follows we shall assume that the radius of the tube R_0 is much longer than the effective radius of the beam, the charge of the beam is completely compensated, the particle losses in the beam are negligibly small, and the frequency of collisions between plasma electrons and the gas is much higher than the electron-ion collision frequency. These assumptions are reasonable for $n_e \gg n_b$ and $n_g \gg n_e\sigma_c/\sigma_0$, where n_b and n_e are the density of electrons in the beam and in the plasma, n_g is the gas density ($n_g \leqq 10^{15}$ cm^{-3}), σ_c is the Coulomb scattering cross section, and σ_0 is the cross section for scattering of electrons by gas atoms. We shall describe the secondary plasma by the hydrodynamic equations, and the primary beam by a system of self-consistent Vlasov equations.

In the kinetic description of a REB, the Bennet distribution is used most often. It can be established, for example, as a result of collisions of electrons in the beam with one another [2] or with particles in the medium [3].

For the distribution function of the electrons in the beam we shall use

$$f = \varkappa \exp\{-H/T + P_z/p_0\}, \tag{1}$$

where $H = c\sqrt{\bar{p}^2 + m^2c^2}$ is the Hamiltonian; $P_z = p_z + eA_z/c$ is the z component of the generalized momentum; $A_{(z)}(r)$ is the z component of the vector potential; $\bar{p}$ is the particle momentum; e and m are the charge and mass; and c is the velocity of light. The equilibrium nature of the state of the beam described is ensured automatically, if the component of the vector potential $A_z(r)$ satisfies Maxwell's equation

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